

# Muon capture by oriented nuclei: new possibilities for studying induced pseudoscalar interaction

A.L.Barabanov, Yu.V.Gaponov, B.V.Danilin, N.B.Shul'gina  
*The Kurchatov Institute, 123182 Moscow, Russia*

## Abstract

Angular distribution of neutrinos (recoil nucleus) in muon capture for an allowed Gamow-Teller transition is considered by taking account of hyperfine effects. This angular distribution is shown to include a correlation of the form  $\sim P_2(\cos \theta)$ , where  $\theta$  is the angle between the neutrino momentum and the axis specifying the orientation of the initial mesic atom. This correlation, which arises only if the initial mesic atom is aligned, proves highly sensitive to the form factor  $g_P$  of induced pseudoscalar interaction. The proposed method for determining  $g_P$  may be realized for the transition  $1^+ \rightarrow 2^+$  from the ground state of the  ${}^6\text{Li}$  nucleus to the narrow resonance of the  ${}^6\text{He}$  nucleus in continuum, as well as in the processes like  ${}^{10}\text{B}(3^+) \rightarrow {}^{10}\text{Be}(2^+)$ ,  ${}^{11}\text{B}(3/2^-) \rightarrow {}^{11}\text{Be}(1/2^-)$ ,  ${}^{23}\text{Na}(3/2^+) \rightarrow {}^{23}\text{Ne}(1/2^+)$  with transitions to bound states.

## 1 Introduction

In the weak nucleon-lepton Hamiltonian, the term that is associated with induced pseudoscalar interaction has received the less study. From the measured rate of muon capture by a free proton, the form factor of this interaction was estimated at  $g_P = 8.7 \pm 1.9$ , which complies with a value of  $8.4 \pm 0.9$  predicted on the basis of the PCAC hypothesis [1]. However, experiments devoted to muon capture by  ${}^6\text{Li}$  [2] and  ${}^{28}\text{Si}$  [3] nuclei yield substantially lower values of  $g_P$ .

In the nonrelativistic limit, the constant  $g_P$  and the constant  $g_M$  of weak magnetism enter into the Hamiltonian in the first order in the ratio of the momentum transfer to the nucleon mass  $M$ . This ratio is of order  $\sim E_\nu/Mc^2$ , where  $E_\nu$  is the neutrino energy, which is about  $\sim 10^{-3}$  in  $\beta$ -decay and about  $\sim 10^{-1}$  in muon capture. Obviously, this ratio controls the sensitivity of the total probabilities of the above processes to  $g_P$ . A detailed analysis of results obtained in [2] for the probability of muon capture by the  ${}^6\text{Li}$  nucleus was carried out in [4] within three-body  $\alpha + 2\text{N}$  model. This analysis was based on the method of hyperspherical functions and employed the expressions derived in [5] for the probability of muon capture. In accord with previous calculations, it was found that the experimental probability of the Gamow-Teller transition from the ground  $1^+$  state of the  ${}^6\text{Li}$  nucleus to the ground  $0^+$  state of the recoil nucleus  ${}^6\text{He}$  is reproduced with the form factor  $g_P$  close to zero.

However, the sensitivity of the muon-capture probability to  $g_P$  is not very high (see above). This brings up the question of whether there are observables in muon capture that are more sensitive to the constant  $g_P$ . In particular, the ratio of the probabilities  $w^+/w^-$  of muon capture from states of hyperfine structure with angular momenta  $F^\pm = J_i \pm 1/2$  is intensively discussed at present as a candidate for such a role [6]-[8]. We sought the answer to this question along different lines, considering the angular distribution of neutrinos (recoil nucleus) in muon capture for an arbitrary Gamow-Teller transition  $J_i \rightarrow J_f = J_i \pm 1$ ,  $\pi_i = \pi_f$ . We performed our analysis in the approximation of only single allowed  $s$ -wave matrix element being taken into account. In the same approximation, the angular distribution of neutrinos was earlier considered in [9] for the particular case of the  $1^+ \rightarrow 0^+$  Gamow-Teller transition.

In calculating the angular distribution of neutrinos in muon capture by a nucleus with a nonzero spin  $J_i$ , we must bear in mind that, when a muon proves to be in the  $1s$ -state from which capture occurs, the state of the nucleus-muon system is a superposition of two states of the hyperfine structure with angular momenta  $F^\pm = J_i \pm 1/2$ . Because of hyperfine interaction, these states are split — that is, they have different energies  $E^\pm$ . The period  $\sim 2\pi\hbar/(E^+ - E^-)$  of oscillations developed by the relative phase of the split states is much less than the characteristic time of muon capture. As a result the interference

of states with different angular momenta  $F$  make no contribution to the differential probability of muon capture. Therefore, the angular distribution of neutrinos is determined by the sum

$$\frac{dw(\mathbf{n}_\nu)}{d\Omega_\nu} = \sum_F P(F) \frac{dw^F(\mathbf{n}_\nu)}{d\Omega_\nu}, \quad (1)$$

where  $P(F)$  is the probability that the state  $|F\rangle$  is populated. Thus, irrespective of whether the processes of capture from states with different  $F$  are detected experimentally or not, the angular distribution of neutrinos must be calculated separately for each state of the hyperfine structure.

It should be emphasized that we mean here the differential probability, involving effects of angular correlations. The total probability contains no contributions from the interference of states with different  $F$ . Therefore, the total probability of muon capture can be calculated without taking hyperfine splitting into account. Hence, it is given by

$$w = \sum_F P(F) w^F. \quad (2)$$

This paper is organized as follows. In Section 2, we consider the polarization features of mesic atoms. Conditions under which the rank-2 spin-tensor of a mesic atom (in other words, the alignment parameter) is nonzero are analyzed. In Section 3, we derive a general expression for the differential probability of muon capture, including effects of angular correlations and taking into account the hyperfine structure of the initial mesic atom. In Section 4, we present the formula for the angular distribution of neutrinos for muon capture from a given state of the hyperfine structure. In this angular distribution, special attention is paid to the term that is proportional to the alignment parameter of the mesic atom. This term exhibits the highest sensitivity to the form factor  $g_P$ . The principal results of this study are summarized in the Conclusion.

## 2 Polarization and alignment of mesic atoms

As was mentioned above, a muon is captured by a nucleus from the  $1s$ -state of a mesic atom. In this state, a muon occurs after a series of transitions. In intermediate states, fine and hyperfine interactions result in a partial transfer of muon polarization to the electron shell and nucleus [5]. It is very difficult to perform accurate calculations for this process. However, this is unnecessary from the viewpoint of practical applications. The residual muon polarization in a given state of the hyperfine structure of the  $1s$ -orbit is reliably determined experimentally from the asymmetry in the electron emission from muon decays.

When the angular momentum  $F$  of a state of the hyperfine structure is higher than  $1/2$ , mesic atoms may display spin-orientation types that are more complicated than mere polarization. Generally, the atom is described by the superposition

$$\Psi_F = \sum_\xi a_\xi(F) \Psi_{F\xi} \quad (3)$$

of states with given projections  $\xi$  of the angular momentum  $F$  onto the  $z$  axis. By performing averaging over the ensemble of atoms, we obtain the density matrix

$$\rho_{\xi\xi'}(F) = \overline{a_\xi(F) a_{\xi'}^*(F)}, \quad \sum_\xi \rho_{\xi\xi}(F) = 1. \quad (4)$$

To describe the polarization of atoms, it is convenient to use the spin-tensors

$$\tau_{Qq}(F) = \sum_{\xi\xi'} C_{F\xi Qq}^{F\xi'} \rho_{\xi\xi'}(F), \quad \tau_{00}(F) = 1. \quad (5)$$

In the case of axially symmetric orientation, we have

$$\tau_{Qq}(F) = \tau_{Q0}(F) \delta_{q0}. \quad (6)$$

The rank-1 and rank-2 spin-tensors are given by

$$\tau_{10}(F) = \frac{\langle \xi \rangle}{(F(F+1))^{1/2}}, \quad \tau_{20}(F) = \frac{3 \langle \xi^2 \rangle - F(F+1)}{((2F-1)F(F+1)(2F+3))^{1/2}}, \quad (7)$$

where  $\langle \xi^n \rangle = \sum_{\xi} \xi^n \rho_{\xi\xi}(F)$ . The rank-1 spin-tensor is proportional to the polarization  $p_1(F) = \langle \xi \rangle / F$ , which is defined in such a way that  $p_1(F) = 1$  only if the state corresponding to the maximum projection  $\xi = F$  is populated. It is convenient to introduce the parameters of orientation of an arbitrary rank  $Q$  that are normalized by the same condition. We specify them as

$$p_Q(F) = \frac{\tau_{Q0}(F)}{(\tau_{Q0}(F))_{\max}}, \quad (\tau_{Q0}(F))_{\max} = C_{FFQ0}^{FF}. \quad (8)$$

For  $Q = 1$  and  $2$ , we then obtain

$$p_1(F) = \left( \frac{F+1}{F} \right)^{1/2} \tau_{10}(F), \quad p_2(F) = \left( \frac{(F+1)(2F+3)}{F(2F-1)} \right)^{1/2} \tau_{20}(F). \quad (9)$$

The density matrix of the ensemble of nonoriented atoms is proportional to identity matrix; hence, we have

$$\tau_{Qq}(F) = \delta_{Q0} \delta_{q0}. \quad (10)$$

Accordingly, the parameters  $\tau_{Q0}(F)$  or  $p_Q(F)$  characterize the orientation of atoms for  $Q \geq 1$ . Under realistic conditions of population, the quantities  $\rho_{\xi\xi}(F)$  smoothly depend on projections  $\xi$ . Therefore, the spin-tensors  $\tau_{Q0}(F)$  or the parameters  $p_Q(F)$  decrease fast with increasing  $Q$ . For this reason, the parameter  $p_2(F)$  is rarely taken into account when the polarization  $p_1(F)$  is nonzero. However, if atoms are oriented in such a way that the populations  $\rho_{\xi\xi}(F)$  of magnetic substates are independent of the sign of  $\xi$ , all odd moments  $\langle \xi \rangle$ ,  $\langle \xi^3 \rangle$ , ... vanish. In this case,  $p_2(F)$  is the first nonzero parameter of orientation. An ensemble of atoms oriented in this way is referred to as an aligned ensemble. Accordingly,  $p_2(F)$  is called the alignment parameter. It is can easily be seen that only atoms with angular momenta  $F > 1/2$  can be aligned. It should be emphasized that the parameter  $p_2(F)$  is nonzero not only when atoms are aligned but also when they are polarized. In accordance with the above definition,  $p_2(F) = 1$  in the case of complete polarization.

In capture of fast muons by atoms, some degree of alignment can appear owing to orbital angular momenta orthogonal to the collision axis [5]. Preorientation of nuclei capturing muons also results in alignment of mesic atoms. Indeed, let a muon with polarization  $p_1(s) = \sqrt{3}\tau_{10}(s)$  directed along the unit vector  $\mathbf{n}_\mu$  proves to be in the  $1s$ -state. Suppose that the nuclear orientation at this instant is determined by the spin-tensors  $\tau_{N0}(I)$  in the reference frame with the  $z$  axis directed along the same vector  $\mathbf{n}_\mu$ . Expanding the direct product of the nuclear and muon wave functions in the functions of states of the hyperfine structure as

$$\sum_m a_m(I) \psi_{Im} \sum_{\sigma} a_{\sigma}(s) \psi_{s\sigma} = \sum_{F\xi} a'_{\xi}(F) \sum_{m\sigma} C_{Im s \sigma}^{F\xi} \psi_{Im} \psi_{s\sigma}, \quad (11)$$

we find that the coefficients in this expansion are given by

$$a'_{\xi}(F) = \sum_{m\sigma} C_{Im s \sigma}^{F\xi} a_m(I) a_{\sigma}(s), \quad \sum_{F\xi} |a'_{\xi}(F)|^2 = 1. \quad (12)$$

The quantity  $P(F) = \sum_{\xi} |a'_{\xi}(F)|^2$  is the probability of finding the mesic atom in the state with angular momentum  $F$ . Taking the square of the equation relating the amplitudes and performing averaging over ensembles, we arrive at a relation between the spin density matrices of nuclei, muons, and mesic atoms. Going over to the spin-tensors, we obtain

$$\begin{aligned} \tau'_{Q0}(F) &= \left( \frac{(2F+1)^3}{(2Q+1)(2s+1)(2I+1)} \right)^{1/2} \times \\ &\times \sum_{NK} (2N+1)(2K+1) \tau_{N0}(I) \tau_{K0}(s) C_{N0K0}^{Q0} \left\{ \begin{matrix} F & I & s \\ F & I & s \\ Q & N & K \end{matrix} \right\}, \end{aligned} \quad (13)$$

where  $\tau'_{00}(F) = P(F)$ , and the symbol  $\{\dots\}$  denotes a 9j-symbol. The normalized spin-tensors of an ensemble of mesic atoms with given  $F$  can be written as

$$\tau_{Q0}(F) = \frac{\tau'_{Q0}(F)}{\tau'_{00}(F)}, \quad \tau_{00}(F) = 1. \quad (14)$$

If nuclei are not oriented ( $\tau_{N0}(I) = \delta_{N0}$ ), the spin-tensors of mesic atoms are given by

$$\tau'_{Q0}(F) = \frac{2F+1}{(2s+1)(2I+1)} U(IsFQ, Fs) \tau_{Q0}(s), \quad (15)$$

where  $U(abcd, ef) = ((2e+1)(2f+1))^{1/2} W(abcd, ef)$  is the normalized Racah function [10]. The populations of states of the hyperfine structure then coincide with the statistical weights

$$P(F) = \frac{2F+1}{(2s+1)(2I+1)}. \quad (16)$$

For  $Q \geq 1$ , the only nonzero parameter among  $p_Q(F)$  is the polarization

$$p_1(F) = \begin{cases} -\frac{1}{3}p_1(s), & F = I - \frac{1}{2}; \\ \frac{2I+3}{3(2I+1)}p_1(s), & F = I + \frac{1}{2}. \end{cases} \quad (17)$$

If  $F = I - \frac{1}{2}$ , mesic atoms possess a nonzero polarization only for  $I \geq 1$ .

It can be seen that oriented nuclei are necessary for mesic atom to be aligned. In this case, it is convenient to single out two contributions in the mesic-atom spin-tensors of even ranks  $Q = 0, 2, \dots$

$$\begin{aligned} \tau'_{Q0}(F) &= \frac{2F+1}{(2s+1)(2I+1)} \left[ U(sIFQ, FI) \tau_{Q0}(I) + \right. \\ &+ 3 \left( \frac{(2F+1)(2s+1)(2I+1)}{2Q+1} \right)^{1/2} \tau_{10}(s) \times \\ &\times \sum_{N=1,3,\dots} (2N+1) \tau_{N0}(I) C_{N010}^{Q0} \left\{ \begin{matrix} F & I & s \\ F & I & s \\ Q & N & 1 \end{matrix} \right\} \left. \right]. \end{aligned} \quad (18)$$

Of these, the second vanishes for unpolarized muons. Because of this second term, populations of states of the hyperfine structure differ from the statistical populations

$$P(F) = \begin{cases} \frac{I}{2I+1} (1 - p_1(s)p_1(I)), & F = I - \frac{1}{2}; \\ \frac{I+1}{2I+1} (1 + \frac{I}{I+1} p_1(s)p_1(I)), & F = I + \frac{1}{2}. \end{cases} \quad (19)$$

It is clear that this effect is insignificant if the polarization of muons is small by the instant at which they prove to be in the  $1s$ -state.

According to (18), the alignment of mesic atoms is determined by the alignment of nuclei, on one hand, and by the fact that nuclear polarization is superimposed on the polarization of muons, on the other hand. The relative sign of these two effects and, hence, the magnitude of alignment depend on the direction of nuclear polarization, provided that the direction of muon polarization is fixed. Setting the level populations equal to their statistical values and disregarding the parameters  $p_N(I)$  of nuclear orientation for  $N \geq 3$ , we find that the alignment of mesic atoms is given by

$$p_2(F) = \begin{cases} p_2(I) - \frac{2}{5}p_1(s)p_1(I), & F = I - \frac{1}{2}; \\ \frac{(2I-1)(I+2)}{(2I+1)(I+1)} \left( p_2(I) + \frac{2(2I+3)}{5(2I-1)} p_1(s)p_1(I) \right), & F = I + \frac{1}{2}. \end{cases} \quad (20)$$

Thus, we see that muon capture by atoms whose nuclei were preoriented — as was the case in experiments reported in [11, 12] — must lead to the formation of aligned mesic atoms. Hence, additional correlations are expected to appear in the angular distribution of neutrinos (recoil nuclei) originating from the decays of mesic atoms aligned as the result of muon capture. Explicit expressions for these correlations are presented in Section 4.

### 3 Muon capture from a state of the hyperfine structure

The Hamiltonian for the muon-capture problem can be taken in the form (see, for example, [5, 13])

$$\hat{H}_W = -\frac{G \cos \theta_C}{\sqrt{2}} \gamma_4(l) \gamma_\lambda(l) (1 + \gamma_5(l)) \hat{\tau}_+(l) \sum_{j=1}^A \delta(\mathbf{r}_j - \mathbf{r}_l) \Gamma_\lambda(j) \hat{\tau}_-(j). \quad (21)$$

where  $G$  is the weak-interaction coupling constant,  $\theta_C$  is the Cabibbo angle, and  $\gamma_\lambda$  matrices are taken in the pseudo-Euclidean metric. This Hamiltonian describes the pointlike interaction of leptons  $l$  with each of the  $A$  intranuclear nucleons numbered by the index  $j$ . The raising  $\hat{\tau}_+(l)$  and lowering  $\hat{\tau}_-(j)$  operators act in the isospin space and transform a muon into a neutrino and a proton into a neutron, respectively.

The operator of the weak nucleon current is given by

$$\Gamma_\lambda = \gamma_4 \left( g_V(k^2) \gamma_\lambda + \frac{\hbar g_M(k^2)}{2M_C} \sigma_{\lambda\rho} k_\rho - g_A(k^2) \gamma_\lambda \gamma_5 - i \frac{\hbar g_P(k^2)}{m_\mu c} k_\lambda \gamma_5 \right). \quad (22)$$

It involves the matrices  $\sigma_{\lambda\rho} = (\gamma_\lambda \gamma_\rho - \gamma_\rho \gamma_\lambda)/2i$ , and the 4-momentum transfer

$$\hbar k_\lambda = p_\lambda - n_\lambda = \nu_\lambda - \mu_\lambda, \quad (23)$$

where  $p_\lambda$ ,  $n_\lambda$ ,  $\mu_\lambda$  and  $\nu_\lambda$  are the 4-momenta of the proton, neutron, muon, and neutrino, respectively. The operator of the weak nucleon current also contains the form factors of vector interaction  $g_V$ , axial-vector interaction  $g_A$ , weak magnetism  $g_M$ , and induced pseudoscalar interaction  $g_P$ . These form factors depend on  $k^2 = k_\lambda k_\lambda$ .

The probability of muon capture is determined according to the Fermi rule. In going over to the nonrelativistic description of intranuclear nucleons, the Hamiltonian of muon capture is subjected to the Foldy-Wouthuysen transformation (see, for example, [13]). Following common practice, we use the transformed Hamiltonian evaluated to first-order terms in  $1/M$ . It should be noted that second-order corrections in  $1/M$  must involve terms associated with nucleon-nucleon potentials.

Let  $\psi_\mu(\mathbf{r}_\mu, \sigma_\mu)$  be the 4-component wave function that describes a muon in the  $1s$ -state with the projection  $\sigma_\mu$  of the spin  $s_\mu = 1/2$  onto the  $z$  axis. Similarly,  $u_\nu(\mathbf{p}_\nu, \sigma_\nu)$  is a 4-spinor that describes a neutrino with momentum  $\mathbf{p}_\nu$  and with the projection  $\sigma_\nu$  of the spin  $s_\nu = 1/2$  onto the  $z$  axis. Following [13], we introduce the 4-vector

$$B_\lambda(\sigma_\mu, \sigma_\nu) = i \overline{\psi_\mu(\mathbf{r}_\mu, \sigma_\mu)} \gamma_\lambda (1 + \gamma_5) u_\nu(\mathbf{p}_\nu, \sigma_\nu). \quad (24)$$

In the nucleon space, the nonrelativistic Hamiltonian of muon capture can then be represented as

$$\begin{aligned} \hat{h}(\sigma_\mu, \sigma_\nu) &= \frac{G \cos \theta_C}{\sqrt{2}} \sum_{j=1}^A \exp\left(-i \frac{\mathbf{p}_\nu \mathbf{r}_j}{\hbar}\right) \times \\ &\times \left\{ -i B_4^+(\sigma_\mu, \sigma_\nu) \left[ g_V(1 + \varepsilon_\nu) + (g_P - g_A) \varepsilon_\nu (\hat{\sigma}_j \mathbf{n}_\nu) + g_A (\hat{\sigma}_j \frac{\hat{\mathbf{p}}_j}{M_C}) \right] + \right. \\ &\left. + \mathbf{B}^+(\sigma_\mu, \sigma_\nu) \left[ g_A \hat{\sigma}_j - i(g_V + g_M) \varepsilon_\nu [\hat{\sigma}_j \times \mathbf{n}_\nu] + g_V \frac{\hat{\mathbf{p}}_j}{M_C} \right] \right\} \hat{\tau}_-(j), \end{aligned} \quad (25)$$

where  $\varepsilon_\nu = E_\nu/2Mc^2$ ,  $E_\nu$  is the neutrino energy, and  $\mathbf{n}_\nu = \mathbf{p}_\nu/p_\nu$  is the unit vector directed along the neutrino momentum. Since weak nucleon-lepton interaction is pointlike, the 4-vectors  $B_\lambda(\sigma_\mu, \sigma_\nu)$  appearing in the Hamiltonian can be taken at  $\mathbf{r}_\mu = \mathbf{r}_j$ . The operators  $\hat{\sigma}_j$  and  $\hat{\mathbf{p}}_j = -i\hbar\partial/\partial\mathbf{r}_j$  act in the space of the  $j$ -th nucleon. The nucleon coordinates  $\mathbf{r}_j$  are reckoned from the center of mass of the nucleus. The above Hamiltonian is a matrix in the space of the projections  $\sigma_\mu$  and  $\sigma_\nu$ . The energy of the emitted neutrino is fixed to be

$$E_\nu = E_f \left[ \left( 1 + \frac{2Q_\mu}{E_f} \right)^{1/2} - 1 \right] \simeq Q_\mu \left( 1 - \frac{Q_\mu}{2E_f} + \dots \right). \quad (26)$$

In this expression,  $Q_\mu = E_i + E_\mu - E_f$ , where  $E_i$  and  $E_f$  are the total energies (including the rest masses) of the initial and final nuclei, respectively, and  $E_\mu$  is the total muon energy, including its binding energy in the atom before capture.

Let  $|J_f M_f\rangle$  be the wave function that describes the internal state of the final nucleus with spin  $J_f$  and its projection  $M_f$  onto the  $z$  axis. At the same time, the initial state of the system involving a nucleus with spin  $J_i$  and a muon with total angular momentum  $F$  is represented as

$$|F\rangle = \sum_{\xi} a_{\xi}(F) \sum_{M_i \sigma_{\mu}} C_{J_i M_i s_{\mu} \sigma_{\mu}}^{F \xi} |J_i M_i\rangle \psi_{\mu}(\sigma_{\mu}). \quad (27)$$

Substituting these functions into the Fermi rule, we find that the differential probability of muon capture per unit time from a given state  $|F\rangle$  of the hyperfine structure is given by

$$\begin{aligned} \frac{dw^F(\mathbf{n}_{\nu})}{d\Omega_{\nu}} &= \frac{1}{(2\pi\hbar^2)^2} \sum_{\sigma_{\nu} M_f} \left| \sum_{\xi} a_{\xi}(F) \sum_{M_i \sigma_{\mu}} C_{J_i M_i s_{\mu} \sigma_{\mu}}^{F \xi} \langle J_f M_f | \hat{h} | J_i M_i \rangle \right|^2 \times \\ &\times \frac{E_{\nu}^2}{c^3(1 + E_{\nu}/E_f)}. \end{aligned} \quad (28)$$

In the nonrelativistic approximation, the muon wave function has the form

$$\psi_{\mu}(\mathbf{r}_{\mu}, \sigma_{\mu}) \simeq \psi_{1s}(r_{\mu}) \begin{pmatrix} \varphi_{\mu}(\sigma_{\mu}) \\ 0 \end{pmatrix}, \quad (29)$$

where  $\varphi_{\mu}(\sigma_{\mu})$  is a conventional two-component spinor. In Hamiltonian (25), the spatial function is taken at  $\mathbf{r}_{\mu} = \mathbf{r}_i$ . Since this function weakly varies over the nuclear volume, its mean value is usually taken outside the matrix-element sign. The square of this mean value is

$$\langle \psi_{1s} \rangle^2 = R(Z) \psi_{1s}^2(0) = \frac{R(Z) Z^3}{\pi} \left( \frac{e^2}{\hbar c} \right)^3 \left( \frac{m'_{\mu} c}{\hbar} \right)^3, \quad (30)$$

where  $Z$  is the charge of the initial nucleus, and  $m'_{\mu} = m_{\mu}/(1 + m_{\mu}c^2/E_i)$  is the reduced mass of the muon-nucleus system. With the aid of the correction factor  $R(Z)$ , it is considered that the nucleus is not a pointlike particle. To write the expression for the probability of muon capture per unit time in more compact form, we introduce the notation

$$A_{\mu} = \lambda_{\mu} \frac{8R(Z)Z^3}{3} \frac{(E_{\nu}/m_{\mu}c^2)^2}{(1 + E_{\nu}/E_f)(1 + m_{\mu}c^2/E_i)^3}, \quad (31)$$

where

$$\lambda_{\mu} = \left( \frac{e^2}{\hbar c} \right)^3 \frac{(G \cos \theta_C)^2 (m_{\mu}c^2)^5}{\hbar^7 c^6} \simeq 1.005 \cdot 10^3 \text{ s}^{-1}. \quad (32)$$

According to (25) and (28), the probability of muon capture is determined by nuclear matrix elements of four types. Each of these is conventionally represented as a multipole expansion (see, for example, [5]). For the Gamow-Teller matrix element, this expansion has the form

$$\begin{aligned} \langle J_f M_f | \sum_{j=1}^A \exp \left( -i \frac{\mathbf{p}_{\nu} \mathbf{r}_j}{\hbar} \right) \hat{\sigma}_{jq} \hat{\tau}_{-}(j) | J_i M_i \rangle &= \\ &= \frac{(4\pi)^{3/2}}{\sqrt{3}} \sum_{wm} (-1)^w Y_{wm}^*(\mathbf{n}_{\nu}) \sum_{uM} C_{1qwm}^u C_{J_i M_i u M}^{J_f M_f} [1wu], \end{aligned} \quad (33)$$

The phase factors  $i^w$ , together with spherical harmonics  $Y_{wm}(\mathbf{r}_j)$ , are conveniently incorporated into the reduced matrix elements  $[1wu]$ . Owing to this, the reduced matrix elements are real-valued quantities, provided that the nuclear wave functions are transformed under the time reversal in the standard way [14]

$$\hat{T} |JM\rangle = (-1)^{J+M} |J-M\rangle. \quad (34)$$

The spherical components  $\hat{\sigma}_{jq}$  of the vector operator  $\hat{\vec{\sigma}}_j$  are defined as  $\hat{\sigma}_{j\pm 1} = \mp(\hat{\sigma}_{jx} \pm i\hat{\sigma}_{jy})/\sqrt{2}$ ,  $\hat{\sigma}_{j0} = \hat{\sigma}_{jz}$ .

## 4 Angular distribution of neutrinos in a Gamow-Teller transition

In this section, we derive the expression for the angular distribution of neutrinos for muon capture by a nucleus with nonzero spin  $J_i$ . In accordance with what was said in the Introduction, we consider an ensemble of mesic atoms in a given state of the hyperfine structure with angular momentum  $F$ . Let the orientation of the ensemble be specified by the parameters  $p_Q(F)$ . The axis of orientation ( $z$  axis) is directed along the unit vector  $\mathbf{n}_\mu$ . We consider muon capture accompanied by the transition of the nucleus into the  $J_f = J_i \pm 1$  state without change in parity ( $\pi_f = \pi_i$ ). We can then expect that the Gamow-Teller matrix element is dominant. According to multipole expansion (33), we have

$$\langle J_f M_f | \sum_{j=1}^A \exp\left(-i \frac{\mathbf{p}_\nu \mathbf{r}_j}{\hbar}\right) \hat{\sigma}_{jq} \hat{\tau}_-(j) | J_i M_i \rangle \simeq \frac{4\pi}{\sqrt{3}} C_{J_i M_i 1 q}^{J_f M_f} [101], \quad (35)$$

that is, we confine our treatment to a single allowed s-wave matrix element. In this approximation, the calculation according to formulas (25) and (28) shows the angular distribution of neutrinos (recoil nuclei) can be represented as

$$\frac{dw^F(\mathbf{n}_\nu)}{d\Omega_\nu} = \frac{A_\mu [101]^2}{4\pi} \frac{2J_f + 1}{2J_i + 1} \left( a_0 + a_1 p_1(F)(\mathbf{n}_\nu \mathbf{n}_\mu) + a_2 p_2(F) \frac{3(\mathbf{n}_\nu \mathbf{n}_\mu)^2 - 1}{2} \right), \quad (36)$$

where

$$a_0 = \left( g_A^2 - \frac{2}{3} g_A (g_P - g_A) \varepsilon_\nu - \frac{4}{3} g_A (g_V + g_M) \varepsilon_\nu \right) C_1(J_i, J_f, F), \quad (37)$$

$$a_1 = - (g_A^2 - 2g_A (g_V + g_M) \varepsilon_\nu) C_2(J_i, J_f, F) - (g_A^2 - g_A (g_P - g_A + g_V + g_M) \varepsilon_\nu) C_3(J_i, J_f, F), \quad (38)$$

$$a_2 = g_A (g_P - g_A - g_V - g_M) \varepsilon_\nu C_4(J_i, J_f, F). \quad (39)$$

This angular distribution is a series in the Legendre polynomials  $P_0(\cos \theta) = 1$ ,  $P_1(\cos \theta) = \cos \theta$ ,  $P_2(\cos \theta) = 3(\cos^2 \theta - 1)/2$ , where  $\theta$  is the angle between the direction of neutrino momentum and the axis of orientation of the initial mesic atom. Since the Hamiltonian of muon capture was taken here in the approximation that is linear in  $\varepsilon_\nu = E_\nu/2Mc^2$ , we retained terms of the same order in the expression for the angular distribution. The coefficients  $C_j(J_i, J_f, F)$  are given by

$$C_1(J_i, J_f, F) = 1 + \sqrt{6} U(F \frac{1}{2} J_i 1, J_i \frac{1}{2}) U(J_f 1 J_i 1, J_i 1), \quad (40)$$

$$C_2(J_i, J_f, F) = \left( \frac{F}{F+1} \right)^{1/2} \left[ \sqrt{2} U\left(\frac{1}{2} J_i F 1, F J_i\right) U(J_f 1 J_i 1, J_i 1) - \sqrt{3} U\left(J_i F \frac{1}{2} 1, \frac{1}{2} F\right) \right], \quad (41)$$

$$C_3(J_i, J_f, F) = 2 \left( \frac{F}{F+1} \right)^{1/2} \left[ \frac{1}{\sqrt{3}} U\left(J_i F \frac{1}{2} 1, \frac{1}{2} F\right) + 5(2(2J_i + 1)(2F + 1))^{1/2} \left\{ \begin{matrix} J_i & F & 1/2 \\ J_i & F & 1/2 \\ 2 & 1 & 1 \end{matrix} \right\} U(J_f 1 J_i 2, J_i 1) \right], \quad (42)$$

$$C_4(J_i, J_f, F) = 2 \left( \frac{10F(2F-1)}{(F+1)(2F+3)} \right)^{1/2} \left[ \frac{1}{3} U\left(\frac{1}{2} J_i F 2, F J_i\right) U(J_f 1 J_i 2, J_i 1) - \right. \\ \left. - ((2J_i+1)(2F+1))^{1/2} \left\{ \begin{matrix} J_i & F & 1/2 \\ J_i & F & 1/2 \\ 1 & 2 & 1 \end{matrix} \right\} U(J_f 1 J_i 1, J_i 1) \right]. \quad (43)$$

The explicit expressions for these coefficients are presented in the Appendix. In the particular case of  $J_i = 1 \rightarrow J_f = 0$ , the formula that we derived for the angular distribution coincides with that presented in [9] to the terms linear in  $\varepsilon_\nu = E_\nu/2Mc^2$  that were taken into account.

It is interesting to note that each of the coefficients  $C_j(J_i, J_f, F)$  vanishes if the initial angular momentum  $F$  of the mesic atom and the final angular momentum  $J_f$  of the nucleus differ by  $3/2$ ; that is,

$$C_j(J_i, J_f = J_i \pm 1, F = J_i \mp \frac{1}{2}) = 0, \quad (44)$$

Obviously, we are dealing here with the trivial consequence of the law of angular-momentum conservation. Indeed, in the approximation of a single  $s$ -wave, the total angular momentum of the system involving a final nucleus and a neutrino can be equal only to  $J_f \pm 1/2$ . Therefore, transitions from states of the hyperfine structure with  $F = J_i \pm 1/2$  into nuclear states with  $J_f = J_i \mp 1$  are forbidden in the approximation that we use.

From expression (36) for the angular distribution, we see that the term proportional to  $\sim P_2(\cos \theta)$ , which describes the anisotropy of neutrino emission in the directions parallel and orthogonal to the vector  $\mathbf{n}_\mu$  and which is due to the alignment of the initial mesic atom, exhibits the highest sensitivity to the form factor  $g_P$  of induced pseudoscalar interaction. This term is in direct proportion to the combination  $(g_P - g_A - g_V - g_M)$ . The reason behind this sensitivity is as follows. In our approximation of a single  $s$ -wave in the exit channel, the dependence of the Hamiltonian on the direction of  $\mathbf{n}_\nu$  is the only source of the angular anisotropy of neutrino emission. The square of the matrix element of Hamiltonian (25) in turn depends on the vector  $\mathbf{n}_\nu$  through two factors. First, the sums of the squared combinations of the components of the 4-vector  $B_\lambda(\sigma_\mu, \sigma_\nu)$  over  $\sigma_\nu$  are linear in  $\mathbf{n}_\nu$ . Second, Hamiltonian (25) involves the product of the vector  $\mathbf{n}_\nu$  and small parameter  $\varepsilon_\nu$ . If the angular distribution is computed to first-order terms in  $\varepsilon_\nu$ , terms that are quadratic in  $\mathbf{n}_\nu$  must be in direct proportion to the difference  $g_P - g_A$  or to the sum  $g_V + g_M$ . But it is the result that we obtained. From these speculations, it follows that correlations that are quadratic in  $\mathbf{n}_\nu$  must exhibit a similar sensitivity to  $g_P$  for mixed Fermi and Gamow-Teller transitions as well.

## 5 Conclusion

In this study, we have considered the angular distribution of neutrinos (recoil nuclei) in muon capture for an arbitrary Gamow-Teller transition, taking into account hyperfine effects. It has been shown that, in the angular distribution, there is a correlation proportional to  $\sim P_2(\cos \theta)$  (where  $\theta$  is the angle between the neutrino momentum and the axis of orientation of the initial mesic atom), which is highly sensitive to the form factor  $g_P$  of induced pseudoscalar interaction — that is, this correlation is in direct proportion to the combination  $(g_P - g_A - g_V - g_M)$ . However, such a correlation appears provided that the initial mesic atom is aligned. Among other things, this means that, for the  $1^+ \rightarrow 0^+$  transition in muon capture by the  ${}^6\text{Li}$  nucleus, the proposed method for studying  $g_P$  is unsuitable if the final nucleus in the above transition is  ${}^6\text{He}$  in the ground state. At least, this is so in the approximation used here, which takes into account the single (leading) reduced matrix element [101]. Indeed, on one hand, the state with angular momentum  $F = 1/2$  cannot be aligned. On the other hand, muon capture from the state with  $F = 3/2$  is forbidden in this approximation. However, this mechanism may prove effective in studying muon capture by the same nucleus  ${}^6\text{Li}$  with the transition into  $J_f^\pi = 2^+$  continuum resonance state of the system involving an  $\alpha$ -particle and two neutrons. The energy and width of this state are  $E = 1.8$  MeV and  $\Gamma = 0.1$  MeV. The neutrino momentum can be reconstructed by measuring the momenta of three particles in the final state. In the case of the  $1^+ \rightarrow 2^+$  transition, muon capture from the  $F = 1/2$  state of the hyperfine structure, which cannot be aligned, is forbidden. Recall that, here, we mean only the approximation of the leading term associated with the reduced matrix element [101]. At  $J_i = 1, J_f = 2$

and  $F = 3/2$ , we obtain  $C_1 = 3/2$ ,  $C_2 = -3/2$ ,  $C_3 = -3/5$ ,  $C_4 = 2/5$  for the coefficients appearing in the angular distribution.

Of course, the proposed method for determining  $g_P$  from the anisotropy of neutrino emission in the directions parallel and orthogonal to the axis of orientation of mesic atoms is applicable to any Gamow-Teller transitions. In particular, it is interesting to investigate muon capture by  $^{10}\text{B}$  ( $3^+ \rightarrow 2^+$ ),  $^{11}\text{B}$  ( $3/2^- \rightarrow 1/2^-$ ),  $^{23}\text{Na}$  ( $3/2^+ \rightarrow 1/2^+$ ), etc., nuclei that is accompanied by transitions to bound states (these processes are discussed in [6, 7]).

This work was supported by the International Science Foundation (grant no. M7C300).

## Appendix

By means of the explicit expressions for the Racah functions [10] and for 9j-symbols [15], the coefficients  $C_j(J_i, J_f, F)$  defined by (40)–(43) can be represented as follows

$$C_1 = \begin{cases} 1 + \frac{J_i(J_i + 1) + 2 - J_f(J_f + 1)}{2J_i}, & F = J_i - \frac{1}{2}; \\ 1 - \frac{J_i(J_i + 1) + 2 - J_f(J_f + 1)}{2(J_i + 1)}, & F = J_i + \frac{1}{2}; \end{cases} \quad (A1)$$

$$C_2 = \begin{cases} \frac{(2J_i - 1)(J_i(J_i + 3) + 2 - J_f(J_f + 1))}{2J_i(2J_i + 1)}, & F = J_i - \frac{1}{2}; \\ \frac{J_i(J_i - 1) - J_f(J_f + 1)}{2(J_i + 1)}, & F = J_i + \frac{1}{2}; \end{cases} \quad (A2)$$

$$C_3 = \begin{cases} \frac{1}{J_i(2J_i + 1)} \left[ (J_i(J_i + 1) + 1 - J_f(J_f + 1))^2 - \right. \\ \left. - J_i(3J_i + 1) - J_f(J_f + 1) + 1 \right], & F = J_i - \frac{1}{2}; \\ \frac{1}{(J_i + 1)(2J_i + 3)} \left[ J_i(3J_i + 5) + J_f(J_f + 1) + 1 - \right. \\ \left. - (J_i(J_i + 1) + 1 - J_f(J_f + 1))^2 \right], & F = J_i + \frac{1}{2}; \end{cases} \quad (A3)$$

$$C_4 = \begin{cases} \frac{J_i - 1}{3J_i(J_i + 1)(2J_i + 1)} \left[ (J_i(J_i + 1) + 2 - J_f(J_f + 1)) \times \right. \\ \left. \times ((J_i + 1)(3J_i + 2) - 3J_f(J_f + 1)) - 8J_i(J_i + 1) \right], & F = J_i - \frac{1}{2}; \\ \frac{1}{3(J_i + 1)(2J_i + 3)} \left[ (J_i(J_i + 1) + 2 - J_f(J_f + 1)) \times \right. \\ \left. \times (J_i(3J_i + 1) - 3J_f(J_f + 1)) - 8J_i(J_i + 1) \right], & F = J_i + \frac{1}{2}. \end{cases} \quad (A4)$$

## References

- [1] N.C.Mukhopadhyay, in: Proceedings of the International Workshop on Low Energy Muon Science — LEMS'93, LA-12698-C (Los Alamos, 1993), p.57.
- [2] J.P.Deutsch, L.Grenacs, P.Igo-Kemenes et al., Phys.Lett., 1968, v.26B, p.315.

- [3] V.Brudanin, V.Egorov, T.Filipova et al., Nucl.Phys., 1995, v.A587, p.577.
- [4] N.B.Shul'guna and B.V.Danilin, Preprint of Kurchatov Inst. of Atomic Energy, Moscow, 1993, no. 5681/2.
- [5] V.V.Balashov, G.Ya.Korenman and R.A.Eramzhyan, Pogloshchenie mesonov atomnymi yadrami (Meson Capture by Nuclei), Moscow: Atomizdat, 1978.
- [6] J.Deutsch, in: Proceedings of the International Workshop on Low Energy Muon Science — LEMS'93, LA-12698-C (Los Alamos, 1993), p.65.
- [7] T.P.Gorringe, J.Bauer, B.L.Johnson et al., Ibid., p.92.
- [8] V.A.Kuz'min, A.A.Ovchinnikova and T.V.Tetereva, Yad. Fiz., 1994, vol. 57, p. 1954 [Phys. At. Nucl. (Engl. Transl.), vol. 57, p. 1881].
- [9] W-Y.P.Hwang, Phys.Rev., 1978, v.C17, p.1799; 1978, v.C18, p.1553.
- [10] H.A.Jahn, Proc.Roy.Soc., 1951, v.A205, p.192.
- [11] Y.Kuno, K.Nagamine, T.Yamazaki, Nucl.Phys, 1987, v.A475, p.615.
- [12] N.R.Newbury, A.S.Barton, P.Bogorad et al., Phys.Rev.Lett., 1991, v.67, p.3219.
- [13] J.M.Eisenberg and W.Greiner, Nuclear Theory, v.2 (North-Holland Publishing Company, Amsterdam-London, 1970).
- [14] A.Bohr and B.R.Mottelson, Nuclear Structure, v.1 (W.A.Benjamin, Inc., New York, Amsterdam, 1969).
- [15] I.I.Sobel'man, Introduction to the theory of atomic spectra, Pergamon Press, Oxford, 1973.